CHAPTER 5.0: CIRCULAR FAILURE ANALYSIS

5.1 Introduction

Circular failure is generally observed in slope of soil, mine dump, weak rock and highly jointed rock mass. It is very important to identify the position of most critical circle in analysis of such failure. Although, field observations may provide valuable clues about the mode of failure (rotational, translational, compound, etc.) and possible position of the slip surface, the centre of the most critical circle can only be found by trial and error. Various slip circles may be analysed and the one yielding the minimum factor of safety can eventually be obtained.

Circular failure of slope can be studied for short and long terms depending on condition and site specific requirements. Short term refers to stability of slope during and immediately after construction. In these cases, there is a little opportunity for drainage to occur. Therefore, the analysis should be carried out in terms of total stress using the undrained strength parameters. However, dissipation of pore water pressure can occur in the long term. Therefore, an effective stress analysis using the drained strength parameters, $c'$, $\phi'$ is carried out under this conditions. The following information is required for the assessment of the stability of a slope against circular failure (Hunt, 1986):

- Location, orientation, and shape of a potential or existing failure,
- Distribution of the materials within and beneath the slope,
- Types of material and their representative shear strength parameters,
- Drainage conditions: drained or undrained,
- Distribution of piezometric levels along the potential failure surface and
- Slope geometry to its full height.

For a total stress analysis, the shear strength parameters, friction $\phi_u=0$ and undrained shear $c_u$ are considered. In contrast to this, the effective strength parameters $c'$ and $\phi'$ are used in conjunction with specified value of pore pressure while making an effective stress analysis. The shear strength of the soil is normally given by the Mohr-Coulomb failure criterion:

$$s=c_u \text{ (for total stress analyses)}$$

$$s=c'+\sigma'tan\phi' \text{ (for effective stress analysis)}$$
5.2 Stability analysis of slope

Most conventional stability analyses of slopes have been made by assuming that the curve of potential sliding is an arc of a circle. The procedures of stability analysis may be divided into two major categories.

1. Mass procedure: In this method, the mass of soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is not the case in most natural slopes.

2. Method of slices: In this procedure, the soil above the surface of sliding is divided into a number of vertical parallel slices. The stability of each slice is calculated separately. This is a versatile technique in which the non-homogeneity of the soil and the pore pressure can be taken into consideration. It also accounts for the variation of the normal stress along the potential failure surface.

Generally the factor of safety is defined as follows

$$FOS = \frac{\text{Shear strength}}{\text{Shear stress}}$$

The shear strength of soil consists of two components: cohesion and friction, and may be written as

$$\tau = c + \sigma \tan \phi$$

Where,

\( \tau = \text{Shear strength, MPa} \)

\( \sigma = \text{Normal stress on the potential failure surface, MPa} \)

\( \phi = \text{Friction angle, Degree} \)
5.3 Stability analysis for Cohesionless Soil

In dry or drained sandy or silty soil, a stable bank can be designed based on the requirement that the bank angle be less than the angle of internal friction of the soil (Sherman, 1973). Usually the design is based on the factor of safety (FOS) that is calculated using the angle of the slope $\theta$ and the angle of internal friction $\phi$. The factor of safety then becomes:

$$FOS = \frac{\tan \phi}{\tan \theta}$$

Numerically, the value of FOS range between 1.0 to 2.0, with a commonly accepted design limit of at least 1.25 for non permanent slopes.

5.4 Total Stress Analysis  (Swedish slip circle method)

In a slope consisting of cohesive soil like clay or silty clay, the slope is also stable if the slope angle greater then friction angle. The soil possesses some cohesion that can be used to design the slope angle even at a steeper value. Slip circle technique is used for this purpose. It is based on the assumption that the rotational angle of sliding during failure is circular. This method requires summing of the moments of all forces acting on the slope, including the gravitational force on the soil mass and the shear stress along the failure surface. If the moments are unbalanced in favour of movement, the slope fails else it will remain stable. The factor of safety can be calculated in this case as the ratio of the moments resisting failure to those causing failure.
Figure 1 shows a trial slip circle, with radius $r$ and the centre of rotation $O$. The weight of the soil of the wedge $w$ is of unit thickness acting through its centroid and the centre of gravity of slip zone is $G$.

![Figure 1: Analysis of a trial slip circle](image)

Considering the moment equilibrium about centre $O$ for length of slip arc $L$, the restoring moment is $C_u L r$ and driving moment is $W x$. Therefore, the factor of safety:

$$FOS = \frac{C_u L r}{W x}$$

This procedure is repeated for other circles in order to find the most critical one having the lowest factor of safety. When a tension crack develops, it reduces the arc length over which the cohesion resists movement and it also applies an additional overturning moment if the crack is filled with water (figure 2).
Figure 2: Analysis of a trial slip circle with tension crack filled with water

Under this condition, the factor of safety is calculated by the formula:

\[ FOS = \frac{C_u L_{a1} r}{Wx + P_w Y} \]

Where,

\( L_{a1} \) = length of slip circle = \( (L - \text{crack length}) \), m

\( C_u \) = Cohesion (undrainded unconsodate), MPa

\( P_w \) = water Pressure = \( 0.5Y_w h_0^2 \), MPa

\( Y_w \) = density of water, KN/m\(^3\)

\( h_0 \) = heigth of water column in crack, m
The forces acting on the deep seated slope prone failure are shown in figure 3. The radius of the arc is \( r \), and the length of the sliding surface is \( L \). The depth of the plane of weakness along which failure occurs is \( h \). The weight of the slope is having two components, \( W_1 \) and \( W_2 \), each of which acts along respective lever arm \( l_1 \) and \( l_2 \), creating moments \( W_1 l_1 \) and \( W_2 l_2 \). The two moments act in opposite directions around the point \( O \), first being a sliding moment and second the resisting moment. In addition, there is another resisting moment due to shear strength of soil.

\[
\text{Figure 3: Failure analysis by slip circle method}
\]

The shear strength is acting along the length of the sliding arc at moment arm length \( r \). the slope is stable when

\[
W_1 l_1 \leq W_2 l_2 + sLr
\]

The factor of safety is

\[
FOS = \frac{W_2 l_2 + sLr}{W_1 l_1}
\]
5.5 Ordinary slip circle method

Slip circle method of circular failure analysis uses the theory of limiting equilibrium. It solves a two-dimensional rigid body stability problem using potential slip surface of circular shape. This method is used to investigate the equilibrium of a soil mass tending to move down the slope under influence of gravity.

The trial slip circle is drawn and the material above the assumed slip surface is divided into a number of vertical strips or slices. In the ordinary slip circle, the forces between slices are neglected and each slice is assumed to act independently as a column of soil of unit thickness and width. The weight of each slice is assumed to act at its centre. The factor of safety is assumed to be the same at all points along the slip surface. The surface with the minimum factor of safety is termed the critical slip surface. Such a critical surface and the corresponding minimum factor of safety represent the most likely sliding surface.

Initially, the moment can be calculated for only one (nth) slice. Later, it can be a summation of all the slices. For one strip, the disturbing moment about centre O (figure 4 & 5) is \( W \alpha \) and the driving moment is \( W \pi \sin \alpha \).

Resisting movement = shear strength x length of slice x radius of slip circle

\[ = s_n L \pi r = (c + \sigma_n \tan \phi) L_n r \]

Where, \( \sigma_n = \frac{W_n \cos \alpha_n}{L_n} \)

Therefore,

\[ FOS = \frac{(cL_n + w_n \cos \alpha_n \tan \phi)}{W_n \sin \alpha_n} \]

Following similar approach, the driving and the resisting forces are calculated separately for all the slices and finally the factor of safety of the slip circle is determined by the expression given below:

\[ FOS = \frac{\sum (cL + w \cos \theta \tan \phi)}{\sum W \sin \alpha} \]
Figure 4: Calculation of factor of safety for $n^{th}$ slice

Figure 5: Dividing the slip circle into vertical slices
Stability analysis using the method of slice can also be explained with the use of figure 6 in which AC is an arc of circle representing the trial failure surface. The soil above the trial failure surface is divided into several vertical slices. Various forces act on typical slice considering its unit length perpendicular to the cross section are shown in the figure 23. In this case the factor of safety can be defined as

$$FOS = \frac{\tau_f}{\tau_d}$$

Where,

$\tau_f =$ average shear strength of soil,

$\tau_d =$ average shear stress developed along the potential surface.

For $n^{th}$ slice considered in the figure 7, $W_n$ is its the weight. The forces $N_r$ and $T_r$ are the normal and the tangential components of reaction, $R$. $P_n$ and $P_{n+1}$ are the normal forces that act on the sides of the slice. Similarly, the shearing forces that act on the sides of the slice are $T_n$ and $T_{n+1}$. It is assumed that the resultants of $P_n$ and $T_n$ are equal in magnitude to the resultants of $P_{n+1}$ and $T_{n+1}$, and that their lines of action coincide.

Figure 6: Geometry of circular failure in slope
Figure 7: Geometry of circular failure in slope

For equilibrium consideration

\[ N_r = W_n \cos \alpha_n \]

The resisting shear force

\[ T_r = \tau_d (\Delta L_n) = \frac{\tau_f (\Delta L_n)}{FOS} = \frac{1}{FOS} \left[ c + \sigma \tan \phi \right] \Delta L_n \]

The normal stress

\[ \sigma = \frac{N_r}{\Delta L_n} = \frac{W_n \cos \alpha_n}{\Delta L_n} \]

Now, for equilibrium of the trial wedge ABC, the moment of the driving force equals to the moment of the resisting force about point O. Thus

\[ \sum_{n=1}^{n=p} W_n r \sin \alpha_n = \sum_{n=1}^{n=p} \frac{1}{FOS} \left[ c + \frac{W_n \cos \alpha_n}{\Delta L_n} \tan \phi \right] (\Delta L_n)(r) \]
\[ FOS = \frac{\sum_{n=1}^{n=p} [c\Delta L_n + W_n \cos \alpha_n \tan \phi]}{\sum_{n=1}^{n=p} [W_n \sin \alpha_n]} \]
5.6 Bishop’s Simplified Method of Slices

Bishop (1955) proposed a more refined solution to the ordinary method of slices. This method is probability is most widely used method for circular failure. When incorporated into a computer program, it yields satisfactory results in most cases. In this method, the effects of forces on the sides of each slice are considered. The forces that act on the \( n \)th slice have been drawn as shown in Figure 7. Here

\[
P_n - P_{n+1} = \Delta P \text{ and } T_n - T_{n+1} = \Delta T
\]

\[
T_r = c_d \Delta L_n + N_r \left( \tan \phi_d \right) = \frac{c_d \Delta L_n}{FOS} + N_r \left( \frac{\tan \phi}{FOS} \right)
\]

If we introduce the factor of safety with respect to cohesion as \( FOS_c \) and that with respect to friction as \( FOS_\phi \) defined as

\[
FOS_c = \frac{c}{c_d}
\]

\[
FOS_\phi = \frac{\tan \phi}{\tan \phi_d}
\]

Where, \( c \) is the cohesion strength, \( \phi \) is the angle of friction, while \( c_d \) and \( \phi_d \) are the cohesion and friction developed along the potential failure surface.

Summing the forces in the vertical direction gives

\[
W_n + \Delta T = N_r \cos \alpha_n + \left[ \frac{N_r \tan \phi}{FOS} + \frac{c_d \Delta L_n}{FOS} \right] \sin \alpha_n
\]

Or

\[
N_r = \frac{W_n + \Delta T - \frac{c_d \Delta L_n}{FOS} \sin \alpha_n}{\cos \alpha_n + \frac{\tan \phi \sin \alpha_n}{FOS}}
\]
For equilibrium of the wedge ABC (Figure 7), taking moment about O gives

\[ \sum_{n=1}^{n=p} W_n r \sin \alpha_n = \sum_{n=1}^{n=p} T_r r \]

Where \( T_r = \frac{1}{FOS} (c + \sigma' \tan \phi) \Delta L_n \)

\[ = \frac{1}{FOS} (c \Delta L_n + N_r \tan \phi) \]

Thus

\[ FOS = \frac{\sum_{n=1}^{n=p} (c \Delta L_n + W_n \tan \phi) + \Delta T \tan \phi + \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n} \]

Where

\[ m_{\alpha(n)} = \cos \alpha_n + \frac{\tan \phi \sin \alpha_n}{FOS} \]

For simplicity, if we assumed \( \Delta T = 0 \), then factor of safety become

\[ FOS = \frac{\sum_{n=1}^{n=p} (c \Delta L_n + W_n \tan \phi) \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n} \]
5.7 General Method of slices

Fredlung and Krahn (1977) have shown that the equations of equilibrium can be formulated quite generally. The formulation is the same for circular and non-circular slip surfaces (Figure 8).

Soil properties: \( c', \phi', \gamma \)

Total normal stress: \( \sigma \)

Shear stress: \( \tau \)

Pore water pressure: \( u \)

Failure criterion: \( s = c' + (\sigma - u) \tan \phi' \)

Mobilized shear strength: \( \tau = s / \text{FOS} \)

\( P = \sigma l; \, T = \tau l \)

\( T = (1 / \text{FOS}) [c'l + (P-ul)\tan\phi'] \)

Figure 8: General method of slice

Resolving the forces vertically:

\[
P \cos \alpha + T \sin \alpha = W - (X_R - X_L)
\]
Rearranging and substituting for T gives:

\[ P = \left[ W - (X_R - X_L) - \left( \frac{1}{FOS} \right) (c'l \sin \alpha - u \tan \phi' \sin \alpha) \right] / m_\alpha \]

Where \( m_\alpha = \cos \alpha [1 + \tan \alpha (\tan \phi / FOS)] \)

Resolving the forces horizontally:

\[ T \cos \alpha - P \sin \alpha + E_R - E_L = 0 \]

Rearranging and substituting for T gives:

\[ E_R - E_L = P \sin \alpha - \left( \frac{1}{FOS} \right) [c'l + (P - u l) \tan \phi'] \cos \alpha \]

Overall moment of equilibrium about O yields:

\[ \sum WD = \sum TR + \sum Pf \]

Rearranging and substituting for T gives:

\[ FOS = \frac{\sum [c'l + (P - u l) \tan \phi'] R}{\sum (Wd - Pf)} \]

For Circular slip surfaces \( f = 0, d = R \sin \alpha, and R = \text{Constant}. So, \]

\[ FOS = \frac{\sum [c'l + (P - u l) \tan \phi']}{\sum W \sin \alpha} \]

Overall force equilibrium:

\[ \sum (E_R - E_L) = 0; \sum (X_R - X_L) = 0. Therefore, \]

\[ \sum ((E_R - E_L)) = \sum Ps \sin \alpha - \left( \frac{1}{FOS} \right) [c'l + (P - u l) \tan \phi'] = 0 \]

\[ F_f = \sum [c'l + (P - u l) \tan \phi' \cos \alpha] / \sum P \sin \alpha \]

In order to solve for \( F_m \) and \( F_f \), \( P \) must be evaluated. To do this following assumptions are made:

\( X_R - X_R = 0 \)  Bishop (1955)

\( (X/E) = \text{Constant} \)  Spencer (1967)

\( (X/E) = f(x) \)  Morgenstern and Price (1965)
In general $F_m = F_f$ and Bishop (1955) showed that $F_m$ is much less sensitive to the assumption about interslice forces than $F_f$. 