CHAPTER 13: SENSITIVITY, PROBABILITY AND RELIABILITY ANALYSIS
13.1 Sensitivity Analysis

Principles of static equilibrium to evaluate the balance of driving and resisting forces are used in limit equilibrium methods. The factor of safety is defined as the resisting forces divided by the driving forces, or alternatively as the shear strength divided by the calculated shear stresses. In traditional slope stability analysis methods analysis, single fixed values (typically, mean values) of representative samples or strength parameters or slope parameters are used. The factor of safety is generally calculated for a slope to assess its stability by using single value of soil properties & slope parameters. The deterministic analysis is unable to account for variation in slope properties and parameters and other variable conditions. In reality, each parameter has a range of values and that value will affect the stability of slope. Therefore, geotechnical properties of Slope parameters have always pose some uncertainties in the simulation. This uncertainty imposes a limit on our confidence in the response or output of the model.

Sensitivity analysis of involves a series of calculations in which each significant parameter is varied systematically over its maximum credible range in order to determine its influence upon the factor of safety. It investigates the robustness of a study when the study includes some form of mathematical modelling. In Sensitivity analysis the factor of safety is calculated using upper & lower bound values for these parameters during critical design. It increased understanding or quantification of the system (e.g. understanding relationships between input and output variables); and when Input is subject to many sources of uncertainty including errors of measurement, absence of information and poor or partial understanding of the driving forces and mechanisms. Good modeling practice requires that the modeler provides an evaluation of the confidence in the model, possibly assessing the uncertainties associated with the modeling process and with the outcome of the model itself. Figure 1 show the identification of sliding and toppling blocks on various field conditions.
Figure 1: Identification of sliding and toppling blocks (a) geometry of block on inclined plane (b) conditions for sliding and toppling of block on an inclined plane.

The sensitivity analysis is able to account for variation in slope properties and different geotechnical conditions. The stability of a slope depends on many factors such as water pressure, slope height, slope angle, shear strength, strength shear joint material etc. These factors not only help in designing the slope but also help in understand the failure mechanism the sensitivity analysis is. It determines the effect of various input parameters on stability of slope under different geo-mining conditions. Sensitivity analysis is an interactive process adopted to simulate slope instability more realistically and determine the influence of the different parameters on the factor of safety. It indicates which input parameters may be critical to the assessment of slope stability, and which input parameters are less important.

In sensitivity analysis, a common approach is that of changing one-factor-at-a-time (OAT), to see what effect this produces on the output. This appears a logical approach as any change observed in the output will unambiguously be due to the single factor changed. Furthermore by changing one factor at a time one can keep all other factors fixed to their central or baseline value. This increases the comparability of the results (all ‘effects’ are computed with reference to the same central point in space) and minimizes the chances of computer program crashes, more likely when several input factors are changed simultaneously. The conventional variation by a fixed percentage of the initial parameter value is problematic for two reasons. However, to carry out sensitivity analyses for more than three parameters is cumbersome, and it is difficult to examine
the relationship between each of the parameters. Consequently, the usual design procedure involves a combination of analysis and judgment in assessing the influence on stability of variability in the design parameters, and then selecting an appropriate factor of safety.

The results will depend on how the initial value was chosen because a small initial value leads to a small variation and a greater initial value to a larger variation. On the one hand, if the model response to parameter variations is nonlinear, then, if the initial parameter value is located nearby the upper or lower bound of the valid parameter range, the variation can lead to inadmissible values beyond the bounds of the range.

Therefore, the parameter variation is considered in which the parameters are not varied by a fixed percentage of the initial value but by a fixed percentage of the valid parameter range. Sensitivity is expressed by a dimensionless index I, which is calculated as the ratio between the relative change of model output and the relative change of a parameter (Figure 2). The sensitivity index (I) as defined by Lenhart\textsuperscript{15} is expressed in equation 1. Sensitivity index is calculated and analysed the effect of input parameter on the factor of safety or stability of internal dump slope.

\[ I = \frac{(\Delta y_2 - y_1)/y_0}{2\Delta x/x_0} \]

Mathematically, the dependence of a variable \( y \) from a parameter \( x \) is expressed by the partial derivative \( \frac{\delta y}{\delta x} \). This expression is numerically approximated by a finite difference: let \( y_0 \) is the model output calculated with an initial value \( y \) of the parameter \( x \). This initial parameter value is varied by \( \pm \Delta x \) yielding \( x_1 = x_0 - \Delta x \) and \( x_2 = x_0 + \Delta x \) with corresponding values \( y_1 \) and \( y_2 \).

The finite approximation of the partial derivative \( \frac{\delta y}{\delta x} \) then is

\[ I' = \frac{y_2 - y_1}{2\Delta x} \]

To get a dimensionless index, \( I' \) has to be normalized. The expression for the sensitivity index I then assumes the form
\[ I' = \frac{(y_2 - y_1)/y_0}{2 \Delta x/x_0} \]

Figure 2: Schematic of the relation between an output variable \( y \) and a parameter \( x \).

The sign of the index shows if the model reacts co-directionally to the input parameter change, i.e. if an increase of the parameter leads to an increase of the output variable and a decrease of the parameter to a decrease of the variable, or inversely.
13.2 Probabilistic Design Methods

Traditional methods use principles of static equilibrium to evaluate the balance of driving and resisting forces. The factor of safety is defined as the resisting forces divided by the driving forces, or alternatively as the shear strength divided by the calculated shear stresses. A factor of safety greater than one indicates a stable slope; a value less than one indicates impending failure.

Probabilistic analysis in slope stability involves source uncertainty. A probabilistic analysis is based on a randomness of the parameters affected by uncertainties. The accuracy of an experimental probability density function depends on the number of observations. Geometrical features of rock discontinuities such as spacing, orientation and persistence can be gathered easily than shear strength features of discontinuities. A probabilistic analysis model requires the knowledge or the reliable estimation of the independence of the random variables or the correlation between random variables.

Many variables are involved in slope stability evaluation and the calculation of the factor of safety. It requires geometrical data, physical data on the geologic materials and their shear-strength parameters (cohesion and angle of internal friction), pore-water pressures, geometry of slope, and the unit weights, water pressure, seismic acceleration and friction angle, etc. Traditional slope stability analysis uses single value for each variable to calculate the factor of safety. The output of a traditional stability analysis is a single-value of factor of safety in deterministic estimate. Single value of the factor of safety approach cannot quantify the probability of failure, associated with a particular design. A probabilistic approach to studying geotechnical issues offers a systematic way to treat uncertainties, especially slope stability.

The variable associated with slope design is uncertain due to many reasons. Therefore, to account for uncertainty the probabilistic method can be used for assessing the stability of slope. There are many source of uncertainty in slope stability analysis. The associated uncertainty varies from analysis to analyses and is case specific. The uncertainties are

1. Site topography
2. Site stratigraphy and variability
3. Geologic origin and characteristics of subsurface materials
4. Groundwater level
5. In-situ soil and/or rock characteristics
6. Engineering properties of rock mass
7. Soil & rock behavior

Parameters such as the angle of friction of rock joints, the uniaxial compressive strength of rock specimens, the inclination and orientation of discontinuities in a rock mass and the measured in situ stresses in the rock surrounding an opening do not have a single fixed value but may assume any number of values. There is no way of predicting exactly what the value of one of these parameters will be at any given location. Hence these parameters are described as random variables.
The arithmetic mean, often referred to as simply the mean or average. Suppose the data 
\{x_1, ..., x_n\}. Then the arithmetic mean $\mu_x$ is defined via the equation as

$$
\mu_x = \frac{1}{n} \sum_{i=1}^{n} x_i
$$

The variance describes the extent of the range of the random variable about the mean and is calculated as

$$
\operatorname{Var}[X] = \frac{1}{n-1} \sum (x_i - \mu_x)^2
$$

The standard deviation ($\sigma$) is given by the positive square root of the variance. A small standard deviation will indicate a tightly clustered data set while a large standard deviation will be found for a data set in which there is a large scatter about the mean.

$$
\sigma = \sqrt{\operatorname{Var}[X]}
$$

The coefficient of variation (COV) is the ratio of the standard deviation to the mean. It is dimensionless and it is a particularly useful measure of uncertainty. A small uncertainty would typically be represented by a COV = 0.05 while considerable uncertainty would be indicated by a COV = 0.25.

$$
\text{Cov} = \frac{\sigma}{\mu_x}
$$

If a pair of random variables (X and Y, for example) depend on each other, the variable X and Y are considered to be correlated, and their covariance is defined by

$$
\operatorname{COV}[X, Y] = \frac{1}{n} \sum (x_i - \mu_x)(y_i - \mu_y)
$$

This covariance is very similar to the variance. If the covariance is normalized by the standard of the X and Y variable, the correlation coefficient, $\rho_{xy}$, may be described by

$$
\rho_{xy} = \frac{\text{Cov}[X, Y]}{\sigma_x \sigma_y}
$$

The correlation coefficient ranges in value from -1 to +1. The case $\rho_{xy} = 1$ indicates a perfect positive, linear correlation between the variable between the variable X and Y. The case $\rho_{xy} = -1$
indicates a negative, or inverse correlation, where high values of Y occur for low values of X. If the two random variable are linearly independent, then $\rho_{xy} = 0$.

This analytical method uses information about the probability distribution of the slope’s characteristics to determine the probability distribution of the output of the analysis. Knowledge of the probability distribution of the output allows the engineer to assess the probability of slope failure.

**Probability density function**

Probability density function (PDF) of a continuous random variable is a function that describes the relative likelihood for this random variable to occur at a given point. The probability density function integral over the entire space is equal to one. The PDF defines the distribution of the random variable and can take many shapes, but the most common ones used in geotechnical applications are the normal and lognormal, although the triangular distribution is also gaining popularity.

The normal distribution extends to infinity in both directions, but this is often not a realistic expression of geotechnical data for which the likely upper and lower bounds of a parameter can be defined. For these conditions, it is appropriate to use the beta distribution which has finite maximum and minimum points, and can be uniform, skewed to the left or right, U-shaped or J-shaped (Harr, 1977). However, where there is little information on the distribution of the data, a simple triangular distribution can be used which are defined by three values: the most likely, and the minimum and maximum values. Figure 3 and 4 show the lognormal and normal probability distribution function with mean and standard deviation respectively.
Normal distribution: The normal or Gaussian distribution is the most common type of probability distribution function. It is generally used for probabilistic studies in geotechnical engineering.
problem of defining a normal distribution is to estimate the values of the governing parameters which are the true mean (μ) and standard deviation (σ). The PDF for the normal distribution with a mean, μ, and deviation, is defined by

\[
f(x; \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right]
\]

The distribution is symmetric about the mean, and the random variable can take on values between -∞ and + ∞.

- Beta distributions are very versatile distributions which can be used to replace almost any of the common distributions and which do not suffer from the extreme value problems discussed above because the domain (range) is bounded by specified values.

- Exponential distributions are sometimes used to define events such as the occurrence of earthquakes or rockbursts or quantities such as the length of joints in a rock mass.

- Lognormal distributions are useful when considering processes such as the crushing of aggregates in which the final particle size results from a number of collisions of particles of many sizes moving in different directions with different velocities.

- Weibull distributions are used to represent the lifetime of devices in reliability studies or the outcome of tests such as point load tests on rock core in which a few very high values may occur.

Sampling techniques: Consider a problem in which the factor of safety depends upon a number of random variables such as the cohesive strength c, the angle of friction φ and the acceleration α due to earthquakes or large blasts.
Probabilistic Slope Stability Analysis Methods

The uncertainties inherent to any project should be recognized. Probabilistic analysis takes into consideration the inherent variability and uncertainties in the analysis parameters. Probabilistic analysis produces a direct estimate of the distribution of either the factor of safety or critical height associated with a design or analysis situation.

There are several probabilistic techniques that can be used to evaluate geotechnical situations. Specifically, for geotechnical analysis, researchers have conducted probabilistic evaluations using: Monte Carlo simulations & Point Estimate Method.

Monte Carlo method

It uses pseudo-random numbers to sample from probability distributions. Large numbers of samples are generated and used to calculate factor of safety. Monte Carlo techniques can be applied to a wide variety of problems involving random behaviour and a number of algorithms are available for generating random Monte Carlo samples from different types of input probability distributions.

The input parameters for a Monte Carlo simulation fall into two categories, the deterministic parameters used for a conventional analysis and the parameters which define the distribution of the input variables. For slope stability analysis the deterministic parameters are:

- Critical Height (H) or Factor of Safety (FS)
- Slope Angle from the Horizontal Plane (β)
- Angle of Friction (φ)
- Cohesion (c)
- Unit Weight (γ)
- Saturated Unit Weight (γ Sat)
- Submerged Unit Weight (γ ‘)

For each of these parameters, Monte Carlo simulation requires definition of the descriptive statistics which define the parameters’ distribution. Depending on the data the descriptive statistics may include:

- Maximum
- Mean
- Minimum
- Standard Deviation
- Coefficient of Variation
In the Monte Carlo simulation, the values for each of the input parameters in the analytical equations are determined by sampling from their respective distributions. The required input values are determined during the simulation based on Latin Hypercubic sampling (LHS).

LHS sampling stratifies the input probability distributions during the simulation process. Monte Carlo Simulation concept specifically designated the use of random sampling procedures for treating deterministic mathematical situations. The foundation of the Monte Carlo gained significance with the development of computers to automate the laborious calculations.

The first step of a Monte Carlo simulation is to identify a deterministic model where multiple input variables are used to estimate an outcome. Step two requires that all variables or parameters be identified. The probability distribution for each independent variable is established for the simulation model, (ie normal, beta, log normal, etc). A random trial process is initiated to establish a probability distribution function for the deterministic situation being modeled. During each pass, a random value from the distribution function for each parameter is selected and entered into the calculation. Numerous solutions are obtained by making multiple passes through the program to obtain a solution for each pass. The appropriate number of passes for an analysis is a function of the number of input parameters, the complexity of the modeled situation, and the desired precision of the output. The final result of a Monte Carlo simulation is a probability distribution of the output parameter.

The component random variables for each calculation are needed from a sample of random values that are based on the selected PDF of the variable. Although these PDFs can take on any shape; the normal, lognormal, analysis beta and uniform distributions are among the most favored for analysis. The Monto Carlo simulation follows a four step process:

1. For each component random variable being considered, select a random value that conforms to the assigned distribution.
2. Calculated the value of the FOS using the adopted performance function and the output values obtained from step1.
3. Repeat steps 1 and 2 many times, storing the FOS result from each must calculation.
4. Use the calculated FOS values from the Monto Carlo simulation to estimate (a) the probability, \( P(F < F_c) \), (b) the sample mean and variance, and (c) the FOS PDF from the histogram.

It should be noted that as each Monto Carlo simulation will use a different sequence of random values, the resulting probabilities, means, variances, and histograms may be slightly different. As the number of trials increases the error will be minimize smaller.

Monte Carlo simulation produces a distribution of factor of safety rather than a single valued. The results of a traditional analysis, using a single value for each input parameter can be compared to
the distribution from the Monte Carlo simulation to determine the relative level of conservatism associated with the conventional design.

**Point Estimate Method**

It is an approximate numerical integration approach to probability modeling. The Generalised Point Estimate Method, can be used for rapid calculation of the mean and standard deviation of a quantity such as a factor of safety which depends upon random behaviour of input variables. To calculate a quantity such as a factor of safety, two point estimates are made at one standard deviation on either side of the mean i.e. \((\mu \pm \sigma)\). The factor of safety is calculated for every possible combination of point estimates, producing \(2^n\) solutions where ‘n’ is the number of random variables involved. The mean and the standard deviation of the factor of safety are then calculated from these \(2^n\) solutions.

There is sometimes reluctance to use probabilistic design when design data is limited and that may not be representative of the population. In these circumstances, it is possible to use subjective assessment techniques that provide reasonably reliable probability values from small samples (Roberds, 1990). The use of probability analysis in design requires that there be generally accepted ranges of probability of failure for different types of structure, as there are for factors of safety. The evaluation of the PEM results in a single number for the sample data. This single value is a representative of the sampled population. Thornton (1994) used the PEM to evaluate the probability of slope failures. Input parameters may be assumed to be normally distributed. A model can be developed to estimate the factor of safety. Thornton recognized that application of this methodology requires criteria to define the acceptable level of risk.
13.3 Reliability Analysis

A probabilistic approach, on the other hand, allows for the systematic analysis of uncertainties and for their inclusion in evaluating slope performance. Important geotechnical parameters such as shear strength parameters and pore water pressures may be used as random variables, each with a probability distribution, rather than deterministic values. Consequently, the factor of safety $F$ of a slope under specified conditions must also be regarded as a random variable with a probability distribution.

If one could compute factors of safety with absolute precision, a value of $F=1.1$ or even $1.01$ would be acceptable. However, the uncertainties involved in computing factors of safety, the computed values of factor of safety are never absolutely precise.

The reliability of a slope ($R$) is an alternative measure of stability that considers explicitly the uncertainties involved in stability analyses. The reliability of a slope is the computed probability that a slope will not fail and is $1.0$ minus the probability of failure:

$$R = 1 - P_f$$

Where $P_f$ is the probability of failure and $R$ is the reliability or probability of no failure. Factors of safety are more widely used than $R$ or $P_f$ to characterize slope stability. Although $R$ and $P_f$ are equally logical measures of stability.

There are two common methods of calculating the reliability of a slope.

1. Margin of safety method
2. Monte Carlo method

The margin of safety is the difference between the resisting and displacing forces, with the slope being unstable if the margin of safety is negative. If the resisting and displacing forces are mathematically defined probability distributions $f_D(r)$ and $f_D(d)$ respectively then it is possible to calculate a third probability distribution for the margin of safety. If the lower limit of the resisting force distribution $f_D(r)$ is less than the upper limit of the displacing force distribution $f_D(d)$. The shaded area in Figure 11 being proportional to the area of the shaded zone. The method of calculating the area of the shaded zone is to calculate the probability density function of the margin of safety: the area of the negative portion of this function is the probability of failure (Figure 5). If the resisting and displacing forces are defined by normal distributions, the margin of safety is also a normal distribution, the mean and standard deviation of which are calculated as follows:

Mean, margin of safety \(= f_D(r) - f_D(d)\)

Standard deviation, margin of safety \(= \sqrt{\sigma_r^2 + \sigma_d^2}\)
where $f_D(r)$ and $f_D(d)$ are the mean values, and $\sigma_r$ and $\sigma_d$ are the standard deviations of the distributions of the resisting and displacing forces respectively.

$$P_f = P((f_D(r) - f_D(d) < 0)$$

Reliability Index

$$R = 1 - P_f$$

Figure 5: Calculation of probability of failure using normal distributions: (a) probability density functions of the resisting force $f_R$ and the displacing force $f_D$ in a slope; and (b) probability density function of difference between resisting and displacing force distributions $f_{D(r-d)}$. 
Monte Carlo analysis is an alternative method of calculating the probability of failure which is more versatile than the margin of safety method. Monte Carlo analysis avoids the integration operations that can become quite complex, and in the case of the beta distribution cannot be solved explicitly. The particular value of Monte Carlo analysis is the ability to work with any mixture of distribution types, and any number of variables, which may or may not be independent of each other.

The principal characteristic of the Monte Carlo method is that of generation a large quantity of random numbers varying between 0 and 1. The examined problem variables are generated by these random numbers in such a way as to respect the assumed probability distribution curves. The component input parameters in a slope stability analysis are modeled as random variables and are used to estimate the PDF of the factor of safety. This PDF is characterized by its mean value, $\mu_F$, and standard deviation, $\sigma_F$. Although the PDF of the FOS, $F$, can take on any shape, it is usually assumed to be either normally or log-normally distributed. The total area under the curve for both the normal and lognormal PDF distributions is always equal to 1.0. If one considers FOS values less than one to represent failure, the shaded areas of these distributions will represent the probability of failure, $p_f$. 
Definition of probability of failure for lognormal and normal PDFs.
Figure 6.59  Comparison of safety for two slopes, where Slope A has a higher reliability index due to its smaller $\sigma_F$ value.
Lognormal distribution will always take on values greater than zero, and as this the same for the factor of safety, this distribution is often considered to be more appropriable. For a lognormal PDF of F, the reliability index, \( \beta \), is defined by (Wolff, 1996)

\[
\beta = \frac{|ln(F_c) - \mu_N|}{\sigma_N}
\]

Where \( F_c \) = the critical factor of safety corresponding to unsatisfactory performance or 1.0 for the case of failure.

\[
\sigma_N = \mu_{NF} = \sqrt{ln \left(1 + V^2_F\right)}
\]

\[
\mu_N = \mu_{NF} = ln(\mu_F) - \frac{1}{2} \frac{\sigma^2}{N}
\]

\[
\mu_F = E[F](\text{mean or expected value of a lognormally distributed FOS})
\]

\[
\sigma_F = \frac{\sigma_F}{\mu_F}(\text{coefficient of variation of FOS})
\]

For the special case of the probability of failure, \( F_c = 1.0 \), and Equation 6.91 simplifies

\[
\beta = \frac{-\mu_N}{\sigma_N} = \frac{ln(\mu_F/\sqrt{1+V^2_F})}{\sqrt{ln(1+V^2_F)}}
\]

The reliability index, \( \beta \), describes safety as it is a measure of the number of standard imparating the best estimate of F, and the prescribed critical value of the

In the PDF of F is presumed to be normal, the reliability index is defined by

\[
\beta = \frac{|F_c - \mu_F|}{\sigma_F}
\]

In the critical factor of safety corresponding to unsatisfactory performance, or 1.0 for the case of failure.

\[
\mu_F = E[F] \quad (\text{mean or expected value of a normally distributed FOS})
\]

\[
\sigma_F = \sqrt{Var[F]} \quad (\text{standard deviation of a normally distributed FOS})
\]
The Monte Carlo method simulation can be carried out following these steps:

1. A deterministic method, such as the limit equilibrium method, is chosen and used to calculate the safety factor or the safety margin as being dependent on parameters of the problem which should be modelled probabilistically. (Example shear strength desility etc).
2. The probability density distribution obtained from experimental data measurements can be constructed in a histogram from.
3. The cumulative distribution is constructed for each random variable probability density distribution. This cumulative curve can be drawn, by dividing, for instance, the variation field of each probabilistic parameter into ten intervals, in such a way that an increasing value between 0 and 1 corresponds to each central value of these intervals.
4. Random values are generated and the correspondent values of the random variables are determined. In the case of the example, random values varying in the 0-1 field are generated and for each generation, the correspondent dip angle is determined.
5. The random variable values obtained by the random generation are used as input data for the determination of the correspondent safety factor with the chosen deterministic method.
6. The operation described in points 4 and 5 are repeated until a stable distribution of the safety factor or safety margin probability density is obtained. A reliable number of generations should be as much as to allow the distribution to be stable according to the designed approximation degree. The distribution stability can be controlled by means of the comparison between two probability density distributions obtained with different generation numbers. The $x^2$ test can be used for this purpose.
Monte Carlo simulation is a computerized mathematical technique that allows to account for variability in their process to enhance quantitative analysis and decision making. The term Monte Carlo was coined in the 1940s by physicists working on nuclear weapon projects in the Los Alamos National Laboratory. Monte Carlo simulation performs variation analysis by building models of possible results by substituting a range of values—a probability distribution—for any factor that has inherent uncertainty. It then calculates results over and over, each time using a different set of random values from the probability functions. Depending on the number of uncertainties and the ranges specified for them, a Monte Carlo simulation could involve thousands or tens of thousands of recalculations before it is complete. Monte Carlo simulation produces distributions of possible outcome values;

The number of trial calculation is given by

\[ n = \left( \frac{100d}{E} \right)^2 \frac{1 - p_F}{p_F} \times m \]

Where \( n \) = minimum number of trials for the Monte Carlo simulation

\( P_F \) = probability of unsatisfactory performance (or failure)

\( E \) = relative percent error in estimating the probability, \( p_F \)

\( m \) = number of component random variables

\( d \) = normal standard deviate according to the following confidence levels:

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>Normal Standard Deviate, d</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>1.282</td>
</tr>
<tr>
<td>90%</td>
<td>1.645</td>
</tr>
<tr>
<td>95%</td>
<td>1.960</td>
</tr>
<tr>
<td>99%</td>
<td>2.576</td>
</tr>
</tbody>
</table>

The relative % error can be calculated as

\[ E = \frac{d \sqrt{m(1 - p_F)}}{np_F} \times 100\% \]
With $E$ computed, the resulting $Pr$ value can be considered to be accurate to within a range of $\pm 0.01E \times Pr$, with the associated level of confidence.

Next, random values of component variable are generated and FOS is calculated using performance function. Uniformly distributed values $R1, R2, \ldots, R_N$ is generated between 0 and 1. With these, random values for a standardized normal distribution ($\mu=0, \sigma=1$) can be generated using Box and Muller method:

$$
N_1 = \sqrt{-2\ln R_1} \cos (2\pi R_2)
$$
$$
N_2 = \sqrt{2\ln R_1} \sin (2\pi R_2)
$$
$$
X_i = \mu_X + N_i \sigma_X
$$

Where, $X_i$ = value taken by the normally distributed random variable
For dependent random variable, one of the pair of independent random numbers, $N_1$ or $N_2$, are modified:

$$
N_2^* = N_1 \rho_{XY} + N_2 \sqrt{1 - \rho_{XY}^2}
$$
$$
x_1 = \mu_X + N_1 \sigma_X
$$
$$
y_1 = \mu_Y + N_2^* \sigma_Y
$$

For each set of component variable, FOS is calculated using performance function.
For a given critical FOS value, probability of unsatisfactory performance is reported as:

$$
P(F < F_c) = \frac{n_c}{n_{total}}
$$

Where $n_c$ = number of FOS values less than $F_c$

$n_{total}$ = total number of trial for the Monte Carlo simulation
There are two slope A and Slope B, and the PDFs are presumed to be lognormally distributed for $F$. Slope A has a mean FOS $\mu_F = 1.3$ and slope B, has a higher mean FOS, $\mu_F = 1.5$. In a deterministic approach, sense, it would appear that Slope B is safer than Slope A. However, the FOS PDF for Slope A has a much smaller standard deviation in comparison to Slope B, which leads to reliability index values of $\beta_A = 1.639$ and $\beta_B = 1.416$ for the critical $F=1.0$ assumption. As Slope A has a greater reliability index, Slope A can be considered safer than Slope B as there is less uncertainty associated with Slope A.

Consider another example of a slope with a conventional deterministic factor of safety $F = 1.1$ and compare with a reliability approach based on a mean value $F = 1.1$ and a standard deviation of $\sigma_F = 0.1$. $F = 1.1$ implies a conventional safety margin of 10% which does not reflect the uncertainties that might be associated with slope performance. The value of $F$ is based on a constant value of each of the parameters on which it depends. The variability and thus the overall uncertainty associated with the value of each parameter is thus ignored in calculating the value of $F$. Therefore it will not represent the reliability of the slope accurately. Consider now a reliability approach within a probabilistic framework. Consider $F$ as a random variable or stochastic parameter with mean value $F = 1.1$ and a standard deviation of 0.1 (this implies a coefficient of variation (c.o.v.) of $\nu_F = 9.1\%$). Then the reliability index from Equation (10.1) above is $\beta = 1$. With a mean $F$ of 1.2 and a standard deviation of 10% (note that now this value implies a c.o.v., $\nu_F = 8.3\%$), the reliability index $\beta = 2$, and the probability of failure $p_F$ is about 2.2%. Thus we note that the reliability index has doubled and the probability of failure has decreased by a factor of about 7.2 (about 86%) although the mean factor of safety increased by only about 9.1%.